

## 1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 02 (Physics Part 2 ,Class XI)
Module Name/Title	Unit 10, Module 5, Dynamics of a Harmonic Oscillator Chapter14, Oscillations
Module Id	keph_201405_eContent
Pre-requisites	Periodic motion, periodic sine and cosine function, simple harmonic motion, phase, mechanical energy of an oscillator
Objectives	After going through this modules learners will be able to : <ul style="list-style-type: none"> <li>• Understand dynamics of oscillations of a loaded spring</li> <li>• Appreciate reasons for oscillatory motion</li> <li>• Relate restoring force, spring constant and periodic time</li> </ul>
Keywords	Dynamics of a simple harmonic oscillator, period time of an oscillator, frequency of an oscillator, restoring force, spring constant, inertia factor

## 2. Development Team

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**1. UNIT SYLLABUS**

**UNIT 10**

**Oscillations and waves**

**Chapter 14 oscillations**

Periodic motion, time period, frequency, displacement as a function of time , periodic functions Simple harmonic motion (S.H.M) and its equation; phase; oscillations of a loaded spring-restoring force and force constant; energy in S.H.M. Kinetic and potential energies; simple pendulum derivation of expression for its time period.

Free, forced and damped oscillations (qualitative ideas only) resonance

**Chapter 15 Waves**

Wave motion transverse and longitudinal waves, speed of wave motion , displacement , relation for a progressive wave, principle of superposition of waves , reflection of waves , standing waves in strings and organ pipes , fundamental mode and harmonics ,beats ,Doppler effect

**2. MODULE-WISE DISTRIBUTION UNIT SYLLABUS**

**15 MODULES**

<b>Module 1</b>	<ul style="list-style-type: none"> <li>● <b>Periodic motion</b></li> <li>● <b>Special vocabulary</b></li> <li>● <b>Time period, frequency,</b></li> <li>● <b>Periodically repeating its path</b></li> <li>● <b>Periodically moving back and forth about a point</b></li> <li>● <b>Mechanical and non-mechanical periodic physical quantities</b></li> </ul>
<b>Module 2</b>	<ul style="list-style-type: none"> <li>● <b>Simple harmonic motion</b></li> <li>● <b>Ideal simple harmonic oscillator</b></li> <li>● <b>Amplitude</b></li> <li>● <b>Comparing periodic motions phase ,</b></li> <li>● <b>Phase difference</b></li> </ul>

	<p><b>Out of phase</b>  <b>In phase</b>  <b>not in phase</b></p>
<b>Module 3</b>	<ul style="list-style-type: none"> <li>• <b>Kinematics of an oscillator</b></li> <li>• <b>Equation of motion</b></li> <li>• <b>Using a periodic function ( sine and cosine functions)</b></li> <li>• <b>Relating periodic motion of a body revolving in a circular path of fixed radius and an Oscillator in SHM</b></li> </ul>
<b>Module 4</b>	<ul style="list-style-type: none"> <li>• <b>Using graphs to understand kinematics of SHM</b></li> <li>• <b>Kinetic energy and potential energy graphs of an oscillator</b></li> <li>• <b>Understanding the relevance of mean position</b></li> <li>• <b>Equation of the graph</b></li> <li>• <b>Reasons why it is parabolic</b></li> </ul>
<b>Module 5</b>	<ul style="list-style-type: none"> <li>• <b>Oscillations of a loaded spring</b></li> <li>• <b>Reasons for oscillation</b></li> <li>• <b>Dynamics of an oscillator</b></li> <li>• <b>Restoring force</b></li> <li>• <b>Spring constant</b></li> <li>• <b>Periodic time spring factor and inertia factor</b></li> </ul>
<b>Module 6</b>	<ul style="list-style-type: none"> <li>• <b>Simple pendulum</b></li> <li>• <b>Oscillating pendulum</b></li> <li>• <b>Expression for time period of a pendulum</b></li> <li>• <b>Time period and effective length of the pendulum</b></li> <li>• <b>Calculation of acceleration due to gravity</b></li> <li>• <b>Factors effecting the periodic time of a pendulum</b></li> <li>• <b>Pendulums as ‘time keepers’ and challenges</b></li> <li>• <b>To study dissipation of energy of a simple pendulum by plotting a graph between square of amplitude and time</b></li> </ul>
<b>Module 7</b>	<ul style="list-style-type: none"> <li>• <b>Using a simple pendulum plot its <math>L-T^2</math> graph and use it to find the effective length of a second’s pendulum</b></li> <li>• <b>To study variation of time period of a simple pendulum of a given length by taking bobs of same size but different masses and interpret the result</b></li> <li>• <b>Using a simple pendulum plot its <math>L-T^2</math> graph and use it to calculate the acceleration due to gravity at a particular place</b></li> </ul>
<b>Module 8</b>	<ul style="list-style-type: none"> <li>• <b>Free vibration natural frequency</b></li> <li>• <b>Forced vibration</b></li> <li>• <b>Resonance</b></li> <li>• <b>To show resonance using a sonometer</b></li> <li>• <b>To show resonance of sound in air at room temperature using a resonance tube apparatus</b></li> </ul>

	<ul style="list-style-type: none"> <li>• Examples of resonance around us</li> </ul>
<b>Module 9</b>	<ul style="list-style-type: none"> <li>• Energy of oscillating source, vibrating source</li> <li>• Propagation of energy</li> <li>• Waves and wave motion</li> <li>• Mechanical and electromagnetic waves</li> <li>• Transverse and longitudinal waves</li> <li>• Speed of waves</li> </ul>
<b>Module 10</b>	<ul style="list-style-type: none"> <li>• Displacement relation for a progressive wave</li> <li>• Wave equation</li> <li>• Superposition of waves</li> </ul>
<b>Module 11</b>	<ul style="list-style-type: none"> <li>• Properties of waves</li> <li>• Reflection</li> <li>• Reflection of mechanical wave at i)rigid and ii)non-rigid boundary</li> <li>• Refraction of waves</li> <li>• Diffraction</li> </ul>
<b>Module 12</b>	<ul style="list-style-type: none"> <li>• Special cases of superposition of waves</li> <li>• Standing waves</li> <li>• Nodes and antinodes</li> <li>• Standing waves in strings</li> <li>• Fundamental and overtones</li> <li>• Relation between fundamental mode and overtone frequencies, harmonics</li> <li>• To study the relation between frequency and length of a given wire under constant tension using sonometer</li> <li>• To study the relation between the length of a given wire and tension for constant frequency using a sonometer</li> </ul>
<b>Module13</b>	<ul style="list-style-type: none"> <li>• Standing waves in pipes closed at one end,</li> <li>• Standing waves in pipes open at both ends</li> <li>• Fundamental and overtones</li> <li>• Relation between fundamental mode and overtone frequencies</li> <li>• Harmonics</li> </ul>
<b>Module 14</b>	<ul style="list-style-type: none"> <li>• Beats</li> <li>• Beat frequency</li> <li>• Frequency of beat</li> <li>• Application of beats</li> </ul>
<b>Module 15</b>	<ul style="list-style-type: none"> <li>• Doppler effect</li> <li>• Application of Doppler effect</li> </ul>

**MODULE 5**

### 3. WORDS YOU MUST KNOW

Let us remember the words we have been using in our study of this physics course

- **Rigid body**: an object for which individual particles continue to be at the same separation over a period of time
- **Point object**: if the position of an object changes by distances much larger than the dimensions of the body the body may be treated as a point object
- **Frame of reference** any reference frame the coordinates(x,y,z), which indicate the change in position of object with time
- **Inertial frame** is a stationary frame of reference or one moving with constant speed
- **Observer** someone who is observing objects
- **Rest** a body is said to be at rest if it does not change its position with surroundings
- **Motion** a body is said to be in motion if it changes its position with respect to its surroundings
  
- **Time elapsed** time interval between any two observations of an object
- **Motion in one dimension**. when the position of an object can be shown by change in any one coordinate out of the three (x, y ,z ), also called motion in a straight line
- **Motion in two dimension** when the position of an object can be shown by changes any two coordinate out of the three (x, y ,z ), also called motion in a plane
- **Motion in three dimension** when the position of an object can be shown by changes in all three coordinate out of the three (x, y ,z )
- **Distance travelled** the distance an object has moved from its starting position SI unit m , this can be zero , or positive
- **Displacement** the distance an object has moved from its starting position moves in a particular direction.SI unit: m, this can be zero, positive or negative  
**For a vibration or oscillation**, the displacement could be mechanical, electrical magnetic. mechanical displacement can be angular or linear.
- **Path length** actual distance is called the path length
- **Position time, distance time , displacement time graph** these graphs are used for showing at a glance the position , distance travelled or displacement versus time elapsed
- **Speed** Rate of change of distance is called speed its SI unit is m/s
- **Average speed** = total path length divided total time taken for the change in position
- **Velocity** Rate of change of position in a particular direction is called velocity, it can be zero , negative and positive , its SI unit is m/s
- **Velocity time graph** - graph showing change in velocity with time , this graph can be obtained from position time graphs
- **Acceleration** Rate of change of speed in a particular direction is called velocity, it can be zero , negative and positive , its SI unit is m/s<sup>2</sup>
- **Acceleration- time graph** : graph showing change in velocity with time , this graph can be obtained from position time graphs
- **Instantaneous velocity**  
 Velocity at any instant of time

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- Instantaneous acceleration

Acceleration at any instant of time

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

- kinematics study of motion without considering the cause of motion

- Oscillation:** one complete to and fro motion about the mean position *Oscillation* refers to any periodic motion of a body moving about the equilibrium position and repeats itself over and over for a period of time.
- Vibration:** It is a to and fro motion about a mean position. the periodic time is small. so we can say oscillations with small periodic time are called vibrations. the displacement from the mean position is also small.
- Frequency:** The number of vibrations / oscillations in unit time.
- Time Period:** Time *period* is the time needed for one complete vibration
- Angular frequency:** a measure of the frequency of an object varying sinusoidally equal to  $2\pi$  times the frequency in cycles per second and expressed in radians per second.
- Inertia:** *Inertia* is the tendency of an object in motion to remain in motion, or an object at rest to remain at rest unless acted upon by a force.
- Sinusoidal: like a  $\sin \theta$  vs  $\theta$**  A sine wave or sinusoid is a curve that describes a smooth periodic oscillation.
- Simple harmonic motion (SHM):** repetitive movement back and forth about an equilibrium(mean) position, so that the maximum displacement on one side of this position is equal to the maximum displacement on the other side. The time interval of each complete vibration is the same.
- Harmonic oscillator:** A *harmonic oscillator* is a *physical* system that, when displaced from equilibrium, experiences a restoring force proportional to the displacement.
- Equation of motion** The equations that relate velocity at an instant of time to acceleration and displacement

$$\begin{aligned} x(t) &= A \cos \omega t, \\ v(t) &= -\omega A \sin \omega t, \\ a(t) &= -\omega^2 A \cos \omega t \end{aligned}$$

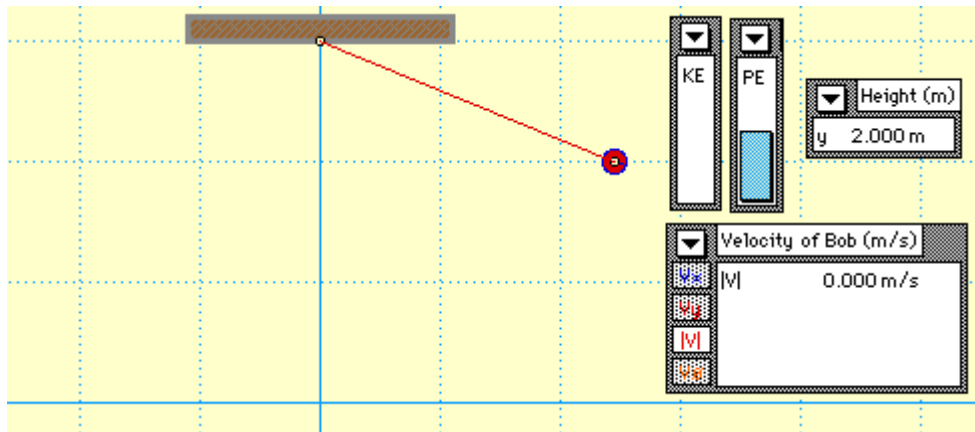
- Mechanical energy:** is the sum of potential **energy** and kinetic **energy**. It is the **energy** associated with the motion and position of an object.
- Restoring force:** is a *force* exerted on a body or a system that tends to move it towards an equilibrium state.
- Conservative force:** is a *force* with the property that the total work done in moving a particle between two points is independent of the taken path. When an object moves from one location to another, the *force* changes the potential energy of the object by an amount that does not depend on the path taken.
- Bob:** A *bob* is the weight on the end of a pendulum
- Periodic motion: motion** repeated in equal intervals of time.
- Amplitude:** the maximum displacement or distance moved by a point on a vibrating body or wave measured from its equilibrium position.

- **Phase:** Phase is the position of a point in time (an instant) on a waveform cycle. In sinusoidal functions or in **waves**, "phase" has two different, but closely related, meanings. One is the initial angle of a sinusoidal function at its origin and is sometimes called **phase difference**.

#### 4. INTRODUCTION

We have already considered **the kinematics of an oscillator**.

**A particle executes simple harmonic motion, if it moves to and fro about a fixed point such that its acceleration at any instant is directly proportional to its displacement at that instant but the direction of acceleration is always towards the mean position**



<http://www.physicsclassroom.com/mmedia/energy/pe.cfm>

Displacement at an instant is given by

$$x(t) = A \cos(\omega t + \phi)$$

The particle velocity and acceleration during SHM as functions of time are given by,

$$v(t) = -\omega A \sin(\omega t + \phi) \text{ (velocity),}$$

$$a(t) = -\omega^2 A \cos(\omega t + \phi)$$

$$= -\omega^2 x(t) \text{ (acceleration),}$$

**We have seen that, both velocity and acceleration of a body executing simple harmonic motion are periodic functions, having the velocity *amplitude*  $v_{\max} = \pm \omega A$  and *acceleration amplitude*  $a_{\max} = \pm \omega^2 A$ , respectively.**

A particle executing simple harmonic motion has, at any time, kinetic energy

$K = \frac{1}{2} mv^2$  and potential energy  $U = \frac{1}{2} kx^2$ . If no friction is present, the mechanical energy of the system,  $E = K + U$  always remains constant even though  $K$  and  $U$  change with time.

**The question is under what condition a particle /object will oscillate? Or what makes a particle execute oscillatory motion?**

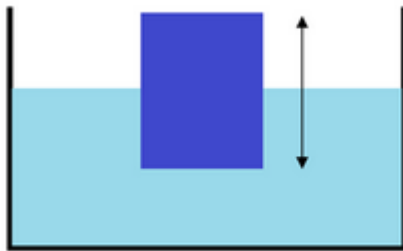
Consider a ball on the ground, if we gently push it, the ball will just move in a straight path, but suppose we take a suitable thread and suspend the ball, a gentle force will cause it to oscillate.

**We will now consider the cause of oscillatory motion**

### 5. SOME SYSTEMS EXECUTING SIMPLE HARMONIC MOTION

There are no physical examples of absolutely pure **simple harmonic motion**. In practice we come across systems that execute nearly simple harmonic motion under certain conditions. In the subsequent part of this section, we discuss the motion executed by some such systems.

#### OSCILLATIONS OF A WOODEN CYLINDER IN WATER



When you push the floating wooden cylinder lower in the water, it tends to bring it back to its original position. A restoring force due to buoyancy will act on it and the wooden cylinder will oscillate up and down in a vertical direction.

Remember that the buoyant force is equal to the weight of the liquid displaced.

In your experience at home a plastic mug will bounce back in a bucket of water

#### THINK ABOUT THIS

A cylindrical piece of cork of density  $\rho$ , base area  $A$  and height  $h$  floats in a liquid of density  $\rho_l$ .

**If the cork is depressed slightly and then released.**

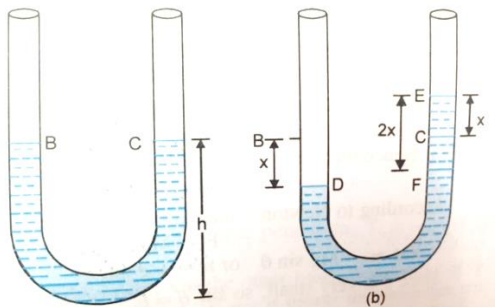
- What would you observe?
- What pushes the cork up?
- Why does it go down again?
- Can the cork oscillate up and down simple harmonically?
- Can we calculate the force that pushes on the cork?



- What will the force depend upon?
- Will it be proportional to the displacement?
- What can you say about the direction of restoring force and the direction of displacement?

## OSCILLATIONS OF A LIQUID COLUMN IN A U TUBE

If a liquid is taken in a U tube, it maintains equal levels in both the columns of the tube  
If we blow on the liquid on side B, the level may go down to say D



### QUESTION

One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. When the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion. Why?

### Observe the gas driven piston

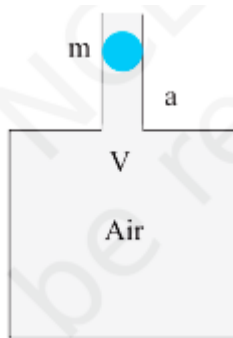
The screenshot shows a YouTube video player with a graph titled "Pressure of gas in cylinder" and "First order binomial expansion of pressure of gas in cylinder". The graph plots pressure (atm) on the y-axis (0.00 to 1.00) against piston position on the x-axis (0.00 to 0.10). A red curve shows the pressure increasing with piston position. Below the graph is a video player showing a piston oscillating. The video title is "Gas-Driven Piston Undergoing Simple Harmonic Oscillation" with 12 views. Below the video is a Wolfram Mathematica link: <http://demonstrations.wolfram.com/Gas...> and a "SUBSCRIBE 7K" button.

<https://www.youtube.com/watch?v=jgimgLUyYhA>

### QUESTION

This demonstration shows simple harmonic oscillation of an isothermal ideal gas in a piston being driven by a pressure gradient. The piston is assumed to be frictionless and

thermal effects of successive expansion and compression of the gas are neglected. This idealized system is a perpetual motion machine! why?

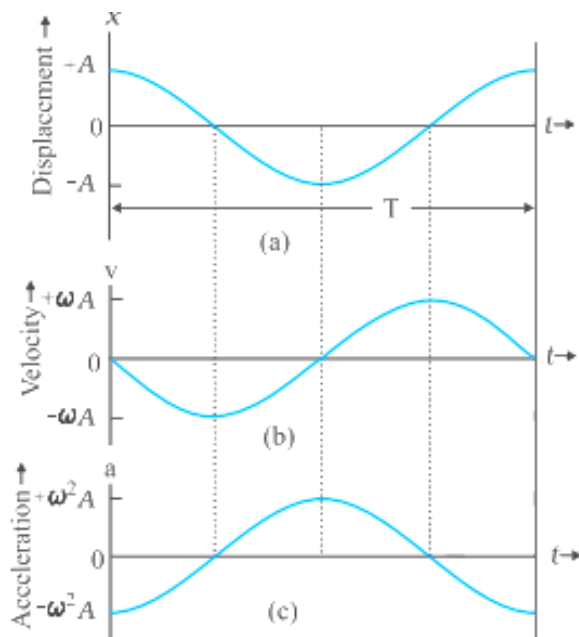


**QUESTION**

An air chamber of volume  $V$  has a neck area of cross section  $a$ , into which a ball of mass  $m$  just fits and can move up and down without any friction. Do you think that when the ball is pressed down a little and released, it executes SHM?

We will now study the **force** responsible for such a motion in which displacement velocity and acceleration vary with time.

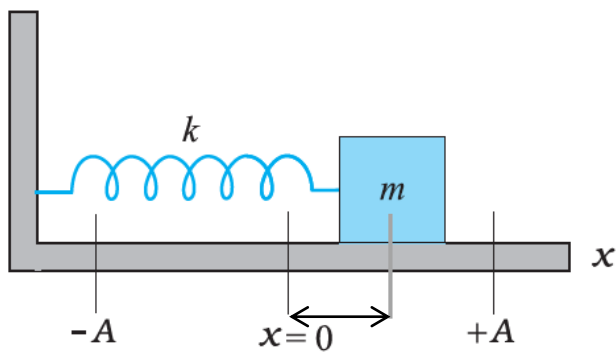
The force changes its direction and has a variable value.



Displacement, velocity and acceleration of a particle in simple harmonic motion have the same period  $T$ , but they differ in phase (graphs not to scale)

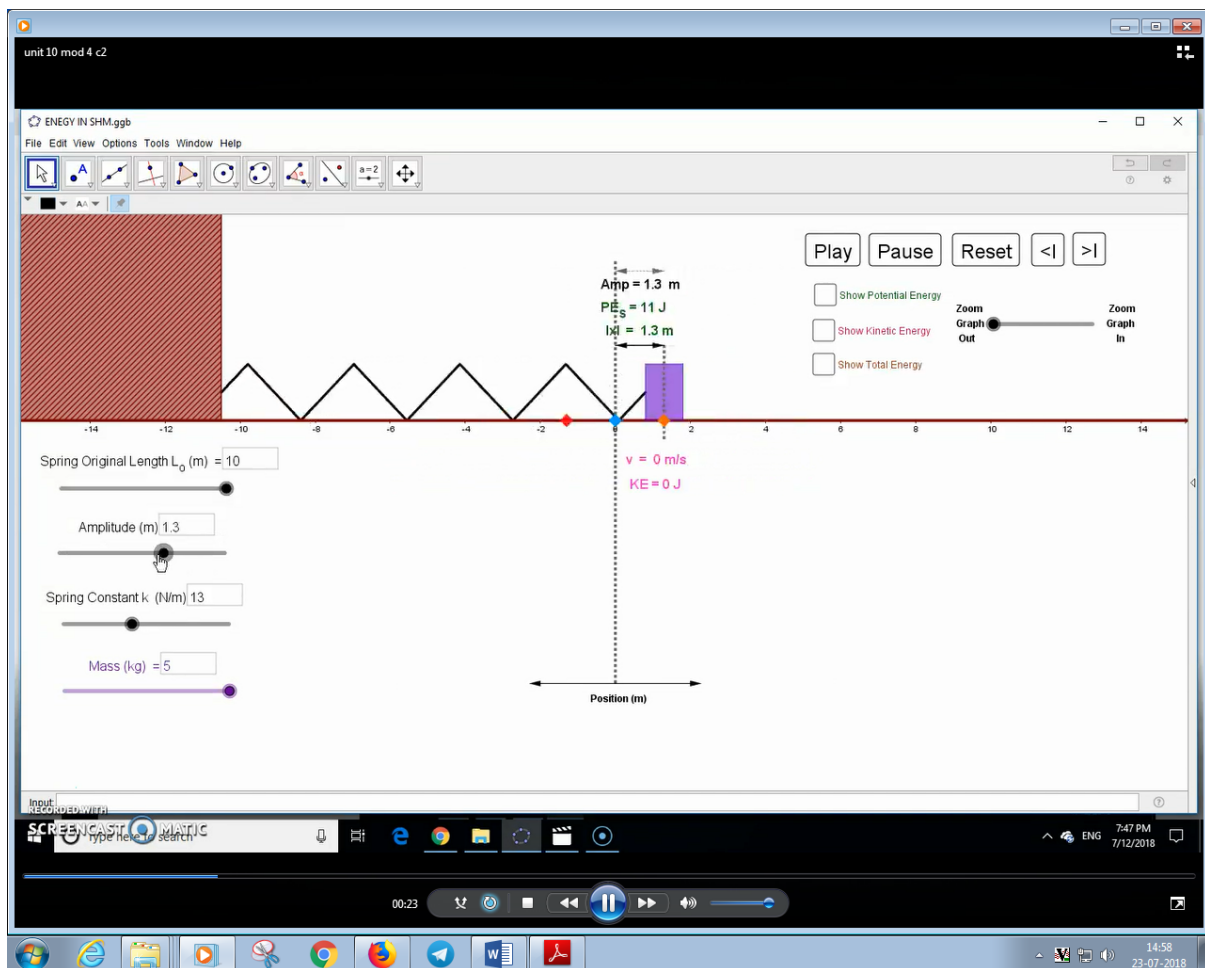
**6. NEED FOR RESTORING FORCE – OSCILLATIONS DUE TO A SPRING**

The simplest example of simple harmonic motion is the oscillations of a block of mass  $m$  attached to a spring, which in turn is fixed to a rigid wall as shown in Figure



A linear simple harmonic oscillator consisting a block of mass  $m$  attached to a spring. The block moves over a frictionless surface. The block, when pulled or pushed and released, executes simple harmonic motion.

Watch the animation



The block is placed on a frictionless horizontal surface.

If the block is pulled on one side and released, it executes a to and fro motion about a mean position.

Let  $x = 0$ , indicate the position of the centre of the block when the spring is in equilibrium.

The positions marked as  $-A$  and  $+A$  indicate the maximum displacements to the left and the right of the mean position.

We have already learnt that springs have special properties, which were first discovered by the English physicist Robert Hooke. He showed that such a system, when deformed, is subject to a restoring force, the magnitude of which is proportional to the deformation or the displacement and acts in opposite direction. This is known as **Hooke's law**.

Hooke's law holds good for displacements small in comparison to the length of the spring.

**At any time, if the displacement of the block from its mean position is  $x$ , the restoring force  $F$  acting on the block is,**

$$F(x) = -kx$$

The constant of proportionality,  $k$ , is called the **spring constant**.

Its value is governed by the elastic properties of the spring. A stiff spring has large  $k$  and a soft spring has small  $k$ .

**The relation between  $F$  and  $x$  is the same as the force law for SHM and therefore the system executes a simple harmonic motion**

### **SALIENT FEATURES OF RESTORING FORCE**

- The force is developed in the body to bring it back to its equilibrium position
- It is opposite to the displacement
- Its value is not constant
- It is proportional to displacement  $F(x) = -kx$
- It is maximum at the extreme position
- It is zero at the mean position
- The factor  $k$  depends upon the oscillating system.
- For a spring it is given by  $F/x$  or force per unit extension

### **EXAMPLE**

A spring balance has a scale that reads 0-50 kg. The length of the scale is 20 cm. What is its spring constant of the spring used in the balance?

### **SOLUTION**

Since the length of the scale is 20 cm, the maximum mass of 50 kg can give 20 cm as extension. In terms of magnitude

$$F = -kx$$

$$50 \times 9.8 = k \times 20 \times 10^{-2}$$

$$k = \frac{50 \times 9.8 \times 100}{20}$$

$$= 2.450 \times 10^3 \text{ N/m}$$

## 7. FORCE LAW FOR SIMPLE HARMONIC MOTION

Using Newton's second law of motion, and the expression for acceleration of a particle undergoing SHM, the force acting on a particle of mass  $m$  in SHM is

$$F(t) = ma = -m\omega^2 \times (t)$$

i.e.,  $F(t) = -kx(t)$

**F(t) is the instantaneous force**

But,

$$F = ma$$

From Newton's second law of motion

$$-kx = ma = -m\omega^2 x$$

This gives,  $k = m\omega^2$

We have,

$$\omega = \sqrt{\frac{k}{m}}$$

and the period,  $T$ , of the oscillator is given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

A block of small mass  $m$  attached to a spring will have, accordingly,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

**Like acceleration, force is always directed towards the mean position - hence it is sometimes called the restoring force in SHM.**

The periodic time can also be expressed as

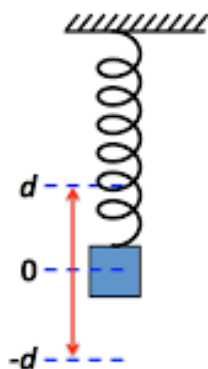
$$T = 2\pi \sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$$

Thus for a body to undergo SHM it should have mass (inertia factor) and a restoring force must set up in the oscillating system (spring factor). Here spring factor does not mean we must have a spring, but it means the system should have a tendency to bring it back to equilibrium position. Examples above can be used to identify the mass which will undergo SHM and the restoring force developed when system shifts from equilibrium condition.

Equilibrium position or condition which we refer as mean position is where the potential energy is the least or minimum.

**EXAMPLE**

Two masses  $m$  and  $2m$  are suspended from similar spring (spring constant is the same for both)



- a) Would the extension for both springs be the same when load is suspended from it?
- b) In which case would the extension be more?
- c) Under what condition would the mass oscillate?
- d) Would the time period be the same?
- e) In which case would the frequency be more? what about the time period?

**SOLUTION**

- a) No
- b) Mass of  $2m$  will extend the spring more
- c) Once the extension is complete and the spring has not elongated beyond the elastic limit, a slight displacement of the mass in the vertical direction will cause the spring to oscillate vertically
- d) No
- e)  $2\pi f = \sqrt{\frac{k}{m}}$  hence  $f$  for smaller mass will be greater, but the time period for mass  $2m$  will be larger

**EXAMPLE**

5 kg collar is attached to a spring of spring constant  $500 \text{ N m}^{-1}$ . It slides without friction over a horizontal rod.

The collar is displaced from its equilibrium position by 10.0 cm and released.

Calculate

- (a) the period of oscillation,
- (b) the maximum speed and
- (c) maximum acceleration of the collar.

**SOLUTION**

a) The period of oscillation is given by

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{5.0\text{kg}}{5000\text{Nm}^{-1}}} \\ = \frac{2\pi}{10} = 0.63\text{s}$$

b) The maximum velocity of the collar executing SHM is given by  $V_{max} = A\omega$

$$= 0.1 \times \sqrt{\frac{k}{m}} = 0.1 \times \sqrt{\frac{500\text{Nm}^{-1}}{5 \text{ kg}}} = 1\text{ms}^{-1}$$

c) The acceleration of the collar at the displacement  $x(t)$  from the equilibrium is given by

$$a(t) = -\omega^2 x(t) = -\frac{k}{m} x(t)$$

Magnitude of  $a_{max}$

$$a_{max} = \omega^2 A = \frac{500\text{Nm}^{-1}}{5\text{kg}} \times 0.1\text{m} = 10\text{ms}^{-2}$$

$a_{max}$  occurs at extreme positions.

**EXAMPLE**

An oscillating spring has a time period of T with mass m. Will T change if

- a) Mass is doubled
- b) Spring is cut into half and the same mass loaded to half the spring

**SOLUTION**

a)  $T = 2\pi \sqrt{\frac{m}{k}}$

The time period will be  $\sqrt{2} T$

b)

$$T' = 2\pi \sqrt{\frac{m}{k'}}$$

Here,  $k' = 2k$

$$T' = 2\pi \sqrt{\frac{m}{2k}}$$

$$T' = \frac{T}{\sqrt{2}}$$

### EXAMPLE

A spring mass system oscillating vertically has a time period of  $T$ . Will this change if the spring vibrates horizontally?

### SOLUTION

The time period remains the same. The time period depends only on  $m$  and  $k$ .

### REDEFINING SHM

A particle executes SHM if it moves to and fro about a mean position such that the acceleration at any instant is directly proportional to its displacement at that instant and is directed towards the mean position.

The restoring force is directed towards the mean position.

### 8. FINDING TIME PERIOD FOR OSCILLATING CYLINDRICAL PIECE OF CORK

The above idea can be used to determine the time period and frequency of any oscillating system. All we have to do is to find the restoring force, find the corresponding spring factor, know the mass of the oscillating system and apply the expression

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Let us consider

- a cylindrical piece of cork of mass  $m$  and
- cross-sectional area  $A$
- floating vertically in a liquid of density  $d$ .

Let  $l$  be the length of the part of the cylinder immersed in the liquid

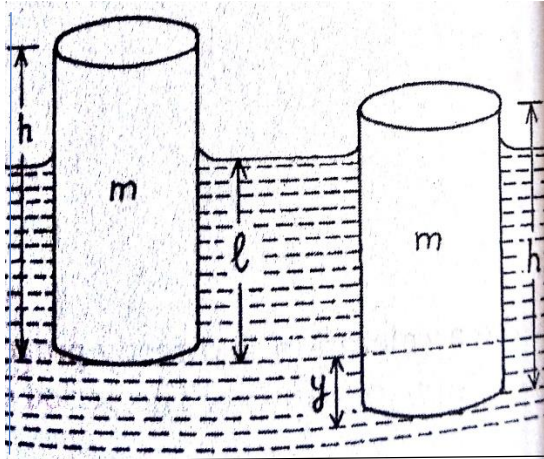
By the law of floatation, the weight of the liquid displaced by the immersed part of the cylinder is equal to the total weight of the cylinder, that is,

$$(Al) dg = mg$$

When the cylinder is pushed down a little into the liquid and left, it begins to oscillate up and down in the liquid.



Suppose, at some instant, the **vertical displacement of the oscillating cylinder from its equilibrium position is  $y$ .**



Then, the **weight (up thrust) of the liquid displaced by the length  $y$  of the cylinder will provide the restoring force  $F$  to the cylinder, that is,**

$$F = -(A y) d g .$$

**Negative sign is taken because the force  $F$  acts in the direction opposite to the displacement of the cylinder.**

If  $\alpha$  is the instantaneous acceleration produced in the motion of the cylinder, then, by Newton's law of motion, we have

$$\alpha = \frac{F}{m} = -\frac{(A) d g}{m} y$$

$$\therefore \alpha = -\omega^2 y,$$

where,

$$\omega = \sqrt{\frac{A d g}{m}} .$$

It follows that the **acceleration  $\alpha$  is proportional to the displacement  $y$  and is directed opposite to it.**

**Hence, the motion of the cylinder is simple harmonic,**

where time-period is given by

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{A d g}} .$$

Mass of the cylindrical body,  $m = \text{volume} \times \text{density} = hA\rho$

Where  $\rho$  is the density of the material of the floating body.

$$\therefore T = 2\pi \sqrt{\frac{hA\rho}{A d g}} = 2\pi \sqrt{\frac{h \rho}{d g}}$$

So if  $d$  is the density of the fluid

$$F = -W(\text{ weight of the displaced fluid}) = -Ax d g$$

Where

$$F = -Axdg$$

$$F = -kx$$

Here the value of **k is Adg.**

A is the area of the circular base of the cylinder and

d is the density of fluid.

**This is the restoring force acting on the cylinder, the negative sign shows that the direction of force is opposite to the direction of displacement.**

You could apply the formula

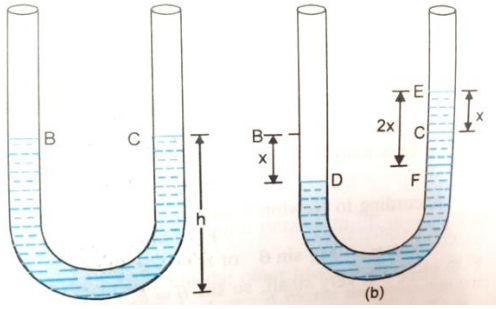
$$T = 2\pi \sqrt{\frac{m}{k}}$$

And get the same result

$$T = 2\pi \sqrt{\frac{m}{Adg}}$$

The time period of a particle executing SHM of small amplitude is independent of amplitude and total energy.

## 9. FINDING TIME PERIOD FOR OSCILLATING LIQUID COLUMN IN A U-TUBE



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mass per unit length =  $m$   
 $\therefore T = 2\pi \sqrt{\frac{m}{\rho g}}$

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## 10. SUMMARY

- The equations of displacement, velocity and acceleration at any instant of an ideal SHM is given by-

$$\begin{aligned}
 x(t) &= A \cos \omega t \\
 v(t) &= -\omega A \sin \omega t \\
 a(t) &= -\omega^2 A \cos \omega t
 \end{aligned}$$

Or

$$\begin{aligned}
 y(t) &= A \sin \omega t \\
 v(t) &= \omega A \cos \omega t \\
 a(t) &= -\omega^2 A \sin \omega t
 \end{aligned}$$

- The magnitude of acceleration of a particle in SHM is the greatest at the end points.
- A particle will execute simple harmonic motion only if a restoring force can set up in it.
- A particle will execute simple harmonic motion if it possess inertia.
- The mechanical energy is conserved in a cyclic process if the forces are conservative.
- Potential energy of an oscillator is given by

$$PE = \frac{1}{2} kx^2$$

Or

$$PE = \frac{1}{2} kA^2 \cos^2(\omega t)$$

- Kinetic energy of an oscillator is given by

$$KE = \frac{1}{2}mv^2$$
$$= \frac{1}{2}m\omega^2A^2\sin^2(\omega t)$$

- The total energy of a particle in SHM equals the potential energy at the end points and the kinetic energy at the mean position.
- The restoring force in SHM is maximum in magnitude when the particle is instantaneously at rest at the extreme position
- The time period of a particle executing SHM of small amplitude is independent of amplitude, phase and total energy.

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